Aspects of CP violation in the HZZ coupling at the LHC

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Abstract

We examine the CP-conserving (CPC) and CP-violating (CPV) effects of a general HZZ coupling through a study of the process $H \to ZZ^{(*)} \to \ell^+\ell^-\ell'^+\ell'^-$ at the LHC. We construct asymmetries that directly probe these couplings. Further, we present complete analytical formulae for the angular distributions of the decay leptons and for some of the asymmetries. Using these we have been able to identify new observables which can provide enhanced sensitivity to the CPV HZZ coupling. We also explore probing CP violation through shapes of distributions in different kinematic variables, which can be used for Higgs bosons with $m_H < 2\,m_Z$.

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1   Introduction

The Standard Model (SM) has had unprecedented success in passing precision tests at the SLC, LEP, HERA and the Tevatron. However, the verification of the Higgs mechanism, which allows the generation of particle masses for fermions and electroweak (EW) gauge bosons without violating the gauge principle, is still lacking. The search for the Higgs boson and the study of its properties will be among the major tasks of the Large Hadron Collider (LHC), which will soon start operation, and of the International Linear Collider (ILC), which is under planning and consideration [1].

However, the instability of the Higgs boson mass to radiative corrections and the resulting fine tuning problem point towards the existence of physics beyond the SM (BSM) at the TeV scale. This BSM physics usually implies more Higgs bosons and may have implications for the properties of the Higgs boson(s). Hence, the determination of the Higgs boson quantum numbers and properties will be crucial to establish it as *the* SM Higgs boson [2] or to probe any new BSM physics.

Furthermore, there is no real theoretical understanding of the relative magnitudes and phases of the different fermion mass parameters in the SM, even though we have an extremely successful description of *all* observed CP-violation (CPV) in terms of the Cabbibo-Kobayashi-Masakawa (CKM) matrix. Indeed, the CPV of the SM, observed only in the K_0–$\bar{K}_0$ and B_0–$\bar{B}_0$ systems to date, appears insufficient to explain the Baryon Asymmetry of the Universe (BAU) [3], and an additional source of CPV beyond that of the SM may be needed for a *quantitative* explanation. An extended Higgs sector together with CPV supersymmetry (SUSY) is one possible BSM option that may explain this BAU [4]. Thus it is clear that the knowledge of the properties of the Higgs sector and any possible CPV therein is of utmost importance in particle physics phenomenology at present [5,6].

The LHC will search for the SM Higgs boson in the entire mass range expected theoretically and still allowed experimentally [7,8], whereas precision profiling of the Higgs boson is expected to be one of the focal points at the ILC [9]. After discovery, the determination of the Higgs boson couplings, in particular those with a pair of electroweak gauge bosons ($V = W/Z$) and those with a pair of heavy fermions ($f = t/\tau$), will be essential. In this study we focus on the HZZ coupling.

The ILC, in both the e^+e^- and the $\gamma\gamma$ [10] options, and the LHC offer a wealth of possibilities for the exploration of the CP quantum numbers of the Higgs boson H [11]. At an e^+e^- collider, the Z boson produced in the process $e^+e^- \to ZH$ is at high energies longitudinally polarised when produced in association with a CP-even Higgs boson and

transversely polarised in case of a CP-odd Higgs boson. The angular distribution of the Z boson therefore carries a footprint of the Higgs boson's CP properties [12–14]. Furthermore, measurements of the threshold excitation curve can yield useful information on the spin and the parity of the Higgs boson and establish it to have spin 0 and be even under parity transformation, hence $J^P = 0^+$, in a model-independent way [15,16]. Additionally, kinematic distributions of the final state particles in the process $e^+e^- \to f\bar{f}H$, produced via vector boson fusion or Higgsstrahlung, where f is a light fermion, with or without initial beam polarisation, can be exploited to study the HZZ coupling, including CPV [13], [17–24]. Ref. [22] uses the optimal observable technique whereas Refs. [19,23,24] exploit the kinematical distributions to construct asymmetries that are directly proportional to different parts of a general CP-violating coupling. Associated production with top quarks $e^+e^- \to t\bar{t}H$ may be used to extract CP information too [25,26].

Higgs decays may also be used effectively. The angular distributions of the Higgs decay products, either a pair of vector bosons or heavy fermions that further decay, can be exploited to gain information on the Higgs CP properties if it is a CP-eigenstate and the CP-mixing if it is CP violating [19], [27–30]. A detailed study of the Higgs spin and parity using the angular distributions of the final-state fermions in $H \to ZZ \to leptons$, above and below the ZZ threshold, was performed in [30]. The $H \to f\bar{f}$ pair $(f = t/\tau)$ has the advantage of being equally sensitive to the CP-even and CP-odd part of the Higgs boson [31]. For Higgs bosons produced in association with heavy fermions, or Higgs decays to heavy fermions at an e^+e^- collider, angular correlations and/or the polarisations of the heavy fermions may also be used [26,32,33].

An ILC operating in the $\gamma\gamma$ mode offers an attractive option not only for the CP-determination of the Higgs boson, but also for the measurement of a small CP-mixing in a state that is dominantly CP-even. Using linear and circular polarisation of the photons one can get a clear measure of the CP mixing [34]; further using a circular beam polarization, the almost mass degenerate CP-odd and CP-even Higgs bosons of the MSSM may be separated [35–39]. Interference effects in the process $\gamma\gamma \to H \to f\bar{f}$ $(f = t/\tau)$ [40–44] can be used to determine the $f\bar{f}H$ and $\gamma\gamma H$ couplings for an H with indefinite CP parity.

Hence, the e^+e^- collider and its possible operation as a $\gamma\gamma$ collider offer some unique possibilities in the exploration of the CP quantum numbers of the Higgs boson. However, the LHC is the next collider to come into operation. So we want to seek answers to these questions already at the LHC [45]. Here, the $t\bar{t}$ final state produced in the decay of an inclusively produced Higgs boson can provide knowledge of the CP nature of the $t\bar{t}H$ coupling through spin-spin correlations [46,47] whereas $t\bar{t}H$ production allows a

determination of the CP-even and CP-odd part of the $f\bar{f}$ couplings with the Higgs boson separately [48,49]. The use of τ polarisation in resonant $\tau^+\tau^-$ production at the LHC has also been recently investigated [50]. The HZZ coupling can be explored at the LHC in the Higgs decay into a Z boson pair which then decay each into a lepton pair, *i.e.* $H \to ZZ^{(*)} \to (\ell^+\ell^-)(\ell'^+\ell'^-)$ [30], [51–53]; above threshold, angular distributions have to be used while below threshold, the dependence on the virtual Z^* boson's invariant mass may be exploited. Furthermore, this coupling (and the HWW coupling) can be studied in vector boson fusion [54–56], and a similar idea may be employed in $H + 2$ jet production [57,58] in gluon fusion (however, also see Ref.[59]).

Most of the suggested measurements should be able to verify a scalar Higgs boson when the full luminosity of $300\,\mathrm{fb}^{-1}$ is collected at the LHC (or even before), provided the Higgs boson is a CP eigenstate. For example, using the threshold behaviour it may be possible to rule out a pure pseudoscalar state with $100\,\mathrm{fb}^{-1}$ in the SM [30]. However, a measurement of the CP mixing is much more difficult, and a combination of several different observables will be essential.

In this paper we investigate CP mixing in the Higgs sector using the process, $H \to ZZ^{(*)} \to (\ell^+\ell^-)(\ell'^+\ell'^-)$. We extend the analysis of Ref. [30] to a Higgs boson of indefinite CP. Further, we extend the analysis of Ref. [53], where asymmetries were constructed using angular distributions of the decay leptons, which directly probe the CP mixing.

The paper is organised as follows. In section 2 we present the complete analytical formulae for the angular distribution of the decay leptons produced in the process $H \to ZZ^{(*)} \to (\ell^+\ell^-)(\ell'^+\ell'^-)$, parameterising the HZZ vertex in a model-independent way, for a Higgs boson of indefinite CP. In section 3 we examine how this modified coupling changes the total number of $H \to ZZ \to 4\,lepton$ events seen at the LHC. In section 4 we then construct different observables that can be used to probe the CP nature of the Higgs boson and present the numerical results. In section 5, we propose an investigation of CP mixing using kinematical distributions of the decay leptons, and in section 6 we present our conclusions.

2 Model independent analysis of $H \to ZZ^{(*)}$

For our study of possible CPV in the Higgs sector we will examine the decay of a Higgs boson into two Z bosons with subsequent decay into two lepton pairs,

$$H \to ZZ^{(*)} \to (f_1 \bar{f}_1)(f_2 \bar{f}_2) . \tag{1}$$

To perform a model-independent analysis we examine the most general vertex including possible CPV for a spin-0 boson[1] coupling to two Z bosons with four-momenta q_1 and q_2, respectively. This can be written as

$$V_{HZZ}^{\mu\nu} = \frac{igm_Z}{\cos\theta_W} \left[a\, g_{\mu\nu} + b\frac{p_\mu p_\nu}{m_Z^2} + c\,\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha k^\beta}{m_Z^2} \right] , \tag{2}$$

where $p = q_1 + q_2$ and $k = q_1 - q_2$, θ_W denotes the weak-mixing angle and $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor with $\epsilon_{0123} = 1$. As can be inferred from Eq. (2) the CP conserving tree-level Standard Model coupling is recovered for $a = 1$ and $b = c = 0$.

The terms containing a and b are associated with the coupling of a CP-even Higgs boson to a pair of Z bosons, while that containing c is associated with that of a CP-odd Higgs boson. In general these parameters can be momentum-dependent form factors that may be generated from loops containing new heavy particles or equivalently from the integration over heavy degrees of freedom giving rise to higher dimensional operators. The form factors b and c may, in general, be complex. Since an overall phase will not affect the observables studied here, we are free to adopt the convention that a is real. This convention requires the assumption that the signal and background do not interfere, and indeed in our approximation where the Higgs boson is taken on-shell, this interference is exactly zero. Interference would be only manifest if the Higgs boson were taken off-shell and since the dominant signal contribution arises from on-shell Higgs bosons, we expect this interference to be small and neglect it.

In principle, the vertex is valid at all orders in perturbation theory. Contributions to the HZZ vertex from loop corrections will not add any new tensor structures and will only alter the values of a, b and c. More generally, a, b and c are momentum dependent form factors obtained from integrating out the new physics at some large scale Λ. Since the momentum dependence will involve ratios of typical momenta in the process to the large scale Λ, we make the reasonable assumption that the scale dependence can be neglected and keep only the constant part.

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[1]In fact, in order to be as general as possible one should allow for a general CP violating coupling with a "Higgs" particle of arbitrary spin, as in [30]. We keep this for future work.

Non-vanishing values for either $\Im m(b)$ or $\Im m(c)$ destroy the hermiticity of the effective theory. Such couplings can be envisaged when going beyond the Born approximation, where they arise from final state interactions, or, in other words out of absorptive parts of the higher order diagrams, presumably mediated by new physics. Further, a, $\Re e(b)$ and $\Im m(c)$ are even under $\tilde{\mathrm{T}}$, while $\Im m(b)$ and $\Re e(c)$ are odd, where $\tilde{\mathrm{T}}$ stands for the pseudo-time reversal transformation, which reverses particle momenta and spins but does not interchange initial and final states. It is the $CP\tilde{\mathrm{T}}$ odd coefficients that are related to the presence of absorptive parts in the amplitude [60]. In most CPV extensions of the SM one has $|a| \gg |b|, |c|$, so most of the observables used to study the HZZ vertex are dominated by the first term in the vertex Eq. (2); in order to probe the last, the CP-odd term, it is most advantageous to construct asymmetries which vanish as CP is restored.

CP violation will be realized if at least one of the CP-even terms is present (i.e. either $a \neq 0$ and/or $b \neq 0$) and c is non-zero. In the following we keep the three coefficients non-zero in our analytical work, where appropriate. However, in the numerical presentation of most of our results we will take $b = 0$ for simplicity, keeping non-zero b only where essential. Further, we make the justified approximation to neglect the possible momentum dependence of the form factors.

Notice that neither $q_{1\mu}V_{HZZ}^{\mu\nu}$ nor $q_{2\nu}V_{HZZ}^{\mu\nu}$ are zero, i.e. the Ward identities are violated. This is due to the breaking of electroweak symmetry and is already the case for the SM vertex. Some studies, e.g. Refs. [24,53], explicitly construct the extra terms such that they satisfy such Ward identities individually, for example, by taking a CP-even term of the form $q_1 \cdot q_2\, g_{\mu\nu} - q_{2\mu}q_{1\nu}$. Strictly speaking, this is not necessary as long as any additional terms vanish in the limit $m_Z \to 0$. Furthermore, since one must separately include the SM $g^{\mu\nu}$ coupling and the new CP-even contribution (with independent coefficients), one may always reproduce our choice of the vertex with a suitable redefinition of the coefficients.

Our vertex differs from the vertex of Refs. [15,30] only in the choice of the normalisation of the coefficients (to make them dimensionless). The normalisation of the coefficients (and the overall normalisation) also differs from Refs. [51,56], where m_H was used in contrast to our m_Z. Additionally, Refs. [51,56] use the momenta of the Z-bosons to define the last term (i.e. $\sim q_1^\alpha q_2^\beta$) in contrast to our $\sim p^\alpha k^\beta$. However, this last difference is for this process only a factor of -2 since the additional terms are removed by the asymmetric property of the tensor. Finally, Ref. [24] differs in the choice of the last term (again $\sim q_1^\alpha q_2^\beta$) and rearranges the contributions of the first two terms, as discussed in the preceding paragraph. For $b = 0$ and light lepton final states, all these vertices are the same, modulo momentum independent normalisations of the coefficients.

From the above discussion it is clear that the total decay rate of Eq. (1), which is CP-even and T̃ even, can only probe a, $\Re e(b)$ and the absolute values of b and c. In order to probe the other non-standard parts of the HZZ coupling, in particular in order to probe CP-violation, one must construct observables that are odd under CP and/or T̃. These observables give rise to various azimuthal and polar asymmetries and will make their presence felt through rates which are integrated over a partial (non-symmetric) phase space. Thus one may probe $\Re e(b)$, $\Im m(b)$, $\Re e(c)$ and $\Im m(c)$ either by using the shapes of various kinematical distributions or by constructing observables which are obtained using partially integrated cross sections [19,23,24][2]. We will use the latter to construct asymmetries which receive contributions from non-standard couplings and which vanish in the tree-level SM. These are related to simple counting experiments, recording the number of events in well defined regions of the phase space. It may also be noted that results obtained using these asymmetries are less sensitive to the effect of radiative corrections to the production [61] and decay [62,63] of the Higgs boson.

In order to find observables which project out the various non-standard couplings in Eq. (2) it is instructive to have an analytical formula for the differential distribution of the Higgs decay to off-shell Z bosons with subsequent decay into fermion pairs with respect to the various scattering angles. We denote the polar angles of the fermions f_1, f_2 in the rest frame of the parent Z bosons by θ_1 and θ_2, and the azimuthal angle between the planes formed from the fermion pairs in the Higgs rest frame by ϕ [see Fig. 1]. Also note that there can be no angular correlations (at tree-level) between the initial and final states (i.e. between the beam-direction and the final state leptons) as long as the Higgs has zero spin.

Introducing the notation $c_{\theta_i} \equiv \cos\theta_i$, $s_{\theta_i} \equiv \sin\theta_i$ $(i = 1,2)$, $c_\phi \equiv \cos\phi$, etc. the tree-level differential decay rate for distinguishable fermions can be cast into the form

$$
\begin{aligned}
\frac{d^3\Gamma}{dc_{\theta_1}\, dc_{\theta_2} d\phi} \sim\ & a^2 \left[s_{\theta_1}^2 s_{\theta_2}^2 - \frac{1}{2\gamma_a} s_{2\theta_1} s_{2\theta_2} c_\phi + \frac{1}{2\gamma_a^2}\left[(1+c_{\theta_1}^2)(1+c_{\theta_2}^2) + s_{\theta_1}^2 s_{\theta_2}^2 c_{2\phi}\right] \right.\\
& \left. - \frac{2\eta_1\eta_2}{\gamma_a}\left(s_{\theta_1} s_{\theta_2} c_\phi - \frac{1}{\gamma_a} c_{\theta_1} c_{\theta_2}\right)\right] \\
+\ & |b|^2 \frac{\gamma_b^4}{\gamma_a^2} x^2\, s_{\theta_1}^2 s_{\theta_2}^2 \\
+\ & |c|^2 \frac{\gamma_b^2}{\gamma_a^2} 4x^2 \left[1 + c_{\theta_1}^2 c_{\theta_2}^2 - \frac{1}{2} s_{\theta_1}^2 s_{\theta_2}^2 (1+c_{2\phi}) + 2\eta_1\eta_2 c_{\theta_1} c_{\theta_2}\right]
\end{aligned}
$$

[2]In fact, Ref. [24] constructed systematically the whole set of asymmetries which probe different parts of the anomalous couplings.
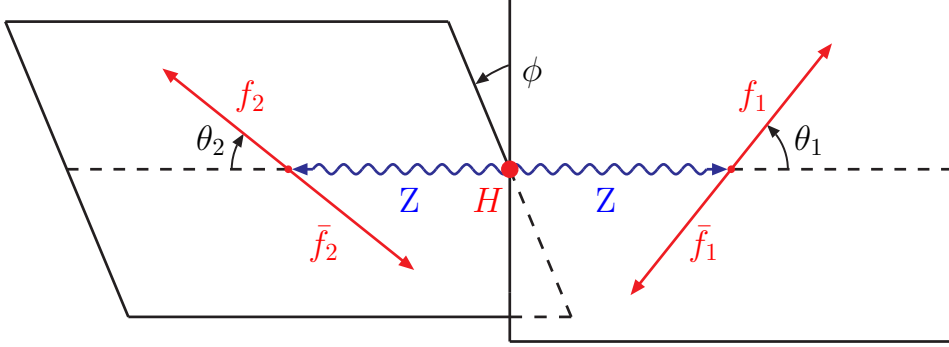
6

Figure 1: The definition of the polar angles θ_i $(i = 1, 2)$ and the azimuthal angle ϕ for the sequential decay $H \to ZZ^{(*)} \to (f_1 \bar{f}_1)(f_2 \bar{f}_2)$.

$$
\begin{aligned}
- \; & 2a\Im m(b)\frac{\gamma_b^2}{\gamma_a^2} x \, s_{\theta_1} s_{\theta_2} s_\phi \left[\eta_2 c_{\theta_1} + \eta_1 c_{\theta_2}\right] \\
- \; & 2a\Re e(b)\frac{\gamma_b^2}{\gamma_a^2} x \left[-\gamma_a s_{\theta_1}^2 s_{\theta_2}^2 + \frac{1}{4} s_{2\theta_1} s_{2\theta_2} c_\phi + \eta_1\eta_2 s_{\theta_1} s_{\theta_2} c_\phi\right] \\
- \; & 2a\Im m(c)\frac{\gamma_b}{\gamma_a} 2x \left[-s_{\theta_1} s_{\theta_2} c_\phi (\eta_1 c_{\theta_2} + \eta_2 c_{\theta_1}) \right. \\
& \left. \qquad\qquad + \frac{1}{\gamma_a}\left(\eta_1 c_{\theta_1}(1 + c_{\theta_2}^2) + \eta_2 c_{\theta_2}(1 + c_{\theta_1}^2)\right)\right] \\
- \; & 2a\Re e(c)\frac{\gamma_b}{\gamma_a} 2x \, s_{\theta_1} s_{\theta_2} s_\phi \left[-c_{\theta_1} c_{\theta_2} + \frac{s_{\theta_1} s_{\theta_2} c_\phi}{\gamma_a} - \eta_1\eta_2\right] \\
+ \; & 2\Im m(b^*c)\frac{\gamma_b^3}{\gamma_a^2} 2x^2 \, s_{\theta_1} s_{\theta_2} c_\phi \left[\eta_2 c_{\theta_1} + \eta_1 c_{\theta_2}\right] \\
+ \; & 2\Re e(b^*c)\frac{\gamma_b^3}{\gamma_a^2} 2x^2 \, s_{\theta_1} s_{\theta_2} s_\phi \left[c_{\theta_1} c_{\theta_2} + \eta_1\eta_2\right] \;,
\end{aligned}
\tag{3}
$$

where $x = m_1 m_2/m_Z^2$ with m_1, m_2 the virtualities of the Z bosons ($q_i^2 = m_i^2$). Furthermore, we have introduced the notation $\gamma_a = \gamma_1\gamma_2(1 + \beta_1\beta_2)$ and $\gamma_b = \gamma_1\gamma_2(\beta_1 + \beta_2)$ in terms of the Lorentz boost factors of the Z bosons, $\gamma_i = 1/\sqrt{1 - \beta_i^2}$, and the velocities

$$
\beta_i = \frac{m_H}{2\,E_i}\beta \qquad i = 1, 2 \;,
\tag{4}
$$

where E_i are the Z boson energies in the Higgs rest frame and

$$
\beta = \left\{\left[1 - \frac{(m_1 + m_2)^2}{m_H^2}\right]\left[1 - \frac{(m_1 - m_2)^2}{m_H^2}\right]\right\}^{1/2} \;.
\tag{5}
$$

The η_i are given in terms of the weak vector and axial couplings v_{f_i}, a_{f_i},

$$
\eta_i = \frac{2\,v_{f_i} a_{f_i}}{v_{f_i}^2 + a_{f_i}^2}, \qquad \text{with} \qquad v_{f_i} = T_{f_i}^3 - 2Q_{f_i}\sin^2\theta_W, \qquad a_{f_i} = T_{f_i}^3 \;.
\tag{6}
$$

Here $T_{f_i}^3$ denotes the third component of the weak isospin and Q_{f_i} the electric charge of the fermion f_i, in our case e^- or μ^-.

3 Sensitivity of the total production to new couplings

As discussed in section 2, one may use the total decay rate of the process in Eq. (1) to test possible deviations from the SM in the Higgs to ZZ coupling. At the LHC the dominant Higgs production process is given by gluon–gluon fusion,

$$gg \rightarrow H \rightarrow ZZ^{(*)} \rightarrow (f_1 \bar{f}_1)(f_2 \bar{f}_2) \,, \tag{7}$$

with $f = e$ or μ. The width for the process $H \rightarrow ZZ^{(*)} \rightarrow (f_1 \bar{f}_1)\,(f_2 \bar{f}_2)$ is given by,

$$\Gamma(H \rightarrow ZZ^{(*)} \rightarrow (f_1 \bar{f}_1)\,(f_2 \bar{f}_2)) =$$
$$\frac{1}{\pi^2} \int_0^{m_H^2} dm_1^2 \int_0^{[m_H - m_1]^2} dm_2^2 \frac{m_Z \, \Gamma_{Z \rightarrow f_1 \bar{f}_1}}{[(m_1^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2]} \frac{m_Z \, \Gamma_{Z \rightarrow f_2 \bar{f}_2}}{[(m_2^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2]} \Gamma_{H \rightarrow ZZ} \,, \tag{8}$$

where the width for the Higgs decay to two Z bosons[3] of virtualities m_1 and m_2 is,

$$\Gamma_{H \rightarrow ZZ} = \frac{G_F m_H^3}{16\sqrt{2}\pi} \beta \left\{ a^2 \left[\beta^2 + \frac{12 m_1^2 m_2^2}{m_H^4} \right] + |b|^2 \frac{m_H^4}{m_Z^4} \frac{\beta^4}{4} + |c|^2 x^2 \, 8\beta^2 \right.$$
$$\left. + a \Re e(b) \frac{m_H^2}{m_Z^2} \beta^2 \sqrt{\beta^2 + 4 m_1^2 m_2^2 / m_H^4} \right\} \tag{9}$$

and $\Gamma_{Z \rightarrow f_i \bar{f}_i}$ is the width for the decay of a Z boson to a fermion pair, $f_i \bar{f}_i$, as given in the SM,

$$\Gamma_{Z \rightarrow f_i \bar{f}_i} = \frac{G_F m_Z^2}{6\sqrt{2}\pi} m_Z \, (v_{f_i}^2 + a_{f_i}^2) \,. \tag{10}$$

As expected, the $\widetilde{CPT}$-even total rate cannot directly test CPV (since there is no interference between the CP-even and CP-odd terms), but it is sensitive to possible non-SM coupling effects in $\Re e(b)$ and the absolute values of b and c. Furthermore, Eq. (9) shows that the linear rise in β just below the threshold is typical [15] of the SM Higgs boson[4].

The Tevatron is in principle also sensitive to the process of Eq. (7) for sufficiently high Higgs boson masses. Indeed, preliminary Tevatron results [65] indicate that a signal for a Higgs boson of 150 GeV would have been seen (with 95% confidence) if the

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[3]For the on-shell decay $H \rightarrow ZZ$, see Ref. [64].

[4]This observation is valid for all spins, with one minor caveat: the spin-2 case can also have a term which presents a linear rise in β but this can be excluded by angular correlations, see Ref. [15].

observed(expected) D0-CDF combined total cross-section were enhanced by a factor of 2.4(3.3). However, this result is dominated by the decay $H \to W^+W^-$; the $H \to ZZ$ decay is suppressed relative to W^+W^- by around a factor of 10 for a 150 GeV Higgs boson, so an enhancement of the HZZ vertex from additional couplings would need to be very large indeed to be seen by the Tevatron. Since we are here investigating the HZZ coupling, we make the assumption that the other decay channels are unaffected and that any change originates from the HZZ coupling alone. For lower Higgs masses the HZZ coupling can also play a role in the production of the Higgs via the channel $q\bar{q} \to Z^* \to ZH$. However, as can be seen from Ref. [66], with current data, the Tevatron would be sensitive to this production mode only if the cross-section were enhanced by a factor of $\sim 30-90$ compared to the SM and thus the nonobservation of this channel in the current data only puts very weak constraints on the magnitude of these couplings.

To estimate the sensitivity of the LHC to deviations from the SM coupling, we refer to the ATLAS study for the process of Eq. (7) at $m_H = 150\,\text{GeV}$ and $200\,\text{GeV}$ [7,45]. In this study, four leptons were selected using the standard electron and muon identification criteria. Events were required to have two leptons with $p_T > 20\,\text{GeV}$ and two additional leptons with $p_T > 7\,\text{GeV}$, with rapidity $|\eta| < 2.5$ for all four. The signal and background were compared in a small mass window around the Higgs boson mass, and a lepton identification and reconstruction efficiency was applied.

For the $m_H = 150\,\text{GeV}$ analysis, one lepton pair was required to have an invariant mass within $10\,\text{GeV}$ of m_Z while the other pair was required to have an invariant mass above $30\,\text{GeV}$. Additionally, isolation and impact parameter cuts were used to further remove irreducible backgrounds. For the $200\,\text{GeV}$ analysis, the continuum ZZ background was further removed by requiring the p_T of the hardest Z-boson to be greater than $m_H/3 \approx 66.6\,\text{GeV}$ (see Refs. [7,45] for further details).

Note that this ATLAS study was performed at tree-level with no K-factors. Higher order corrections to the production process could alter the cross section by up to a factor two [61]. The higher order electroweak corrections to the Higgs decays into W/Z bosons have been calculated in Ref. [62] in the narrow width approximation. Ref. [63] presents the complete $\mathcal{O}(\alpha)$ corrections to the general $H \to 4l$ processes, including off-shell gauge bosons which are important for our study. The corrections have been shown to change the partial width by up to 5% for the Higgs boson masses we consider in this paper. Our analysis, which uses the results of the ATLAS study, strictly speaking is only valid at tree-level, despite the all-orders validity of the HZZ coupling (see section 2).

After these cuts, the study found for a 150 GeV Higgs boson and an integrated lumi-

nosity of 100 fb^{-1}, 67.6 signal events with a background of 8.92 events. The corresponding signal and background events for a 200 GeV Higgs boson were 54 and 7, for an integrated luminosity of 30 fb^{-1}. Altered HZZ couplings will enhance (or decrease) the number of signal events, while leaving the number of background events fixed. However, the size of this enhancement (or reduction) is model-dependent. Although the change in the width for $H \rightarrow ZZ^* \rightarrow 4l$ is clear from Eq. (9), the *branching ratio* depends on how the other Higgs decay channels are affected by the new physics. As mentioned above, we here make the assumption that only the HZZ vertex deviates from that of the SM. If this were not the case, and, for example, the HWW coupling was similarly enhanced, then any enhancement of the $H \rightarrow ZZ$ branching ratio would be watered down. Furthermore, we assume the Higgs production proceeds as according to the SM, since the dominant production mode contains no HZZ coupling, but one should be aware that CPV effects in other vertices may alter the Higgs production rate (see e.g. Ref. [67]). Finally, we assume that the rate calculated with the general HZZ coupling Eq. (2) will be reduced by experimental cuts in the same way as the SM rate. Only electron and muon final states are considered, and we scale up the number of signal and background events to correspond to an integrated luminosity of 300 fb^{-1}.

We then calculate the total number of signal events N_S that we expect from the new coupling and compare the expected change (with respect to the SM) with the possible statistical fluctuations of the SM signal and backgrounds. The significance of this deviation from the SM expectation (in units of one standard deviation) is then $(N_S - N_S^{\mathrm{SM}})/\sqrt{N_S^{\mathrm{SM}} + N_B}$, where N_S^{SM} is the number of signal events expected in the SM and N_B is the number of background events. This quantity, for $m_H = 150\,\mathrm{GeV}$ and $200\,\mathrm{GeV}$, is plotted in Fig. 2, where we have scanned over values of the couplings a and $|c|$ (the total rate is independent of the phase of c). For simplicity, we have set $b = 0$ (for $b = 0$, one can see that Eq. (9) is symmetric in a allowing us to restrict the plot to positive values). As can be inferred from Fig. 2 in the white region we can not distinguish the corresponding a, c values from the SM case, $a = 1$, $c = 0$ at a significance more than 3σ.

Large values of $|c|$, however, together with the SM value of $a = 1$ are easily identified at the LHC. For example, the scenario $a = c = 1$ is excluded with around 5σ significance for $m_H = 150\,\mathrm{GeV}$ and over 20σ significance at $m_H = 200\,\mathrm{GeV}$. However, since $|c|$ arises from new physics one would expect its value to be suppressed by the size of the new physics scale, and therefore be rather small. For $a = 1$ (the SM value) we find that this measurement provides 3σ evidence of non-zero c only if $c \gtrsim 0.75$ or $c \gtrsim 0.32$, for $m_H = 150\,\mathrm{GeV}$ and $200\,\mathrm{GeV}$, respectively. Furthermore, since both a^2 and $|c|^2$ contribute to the total rate, we
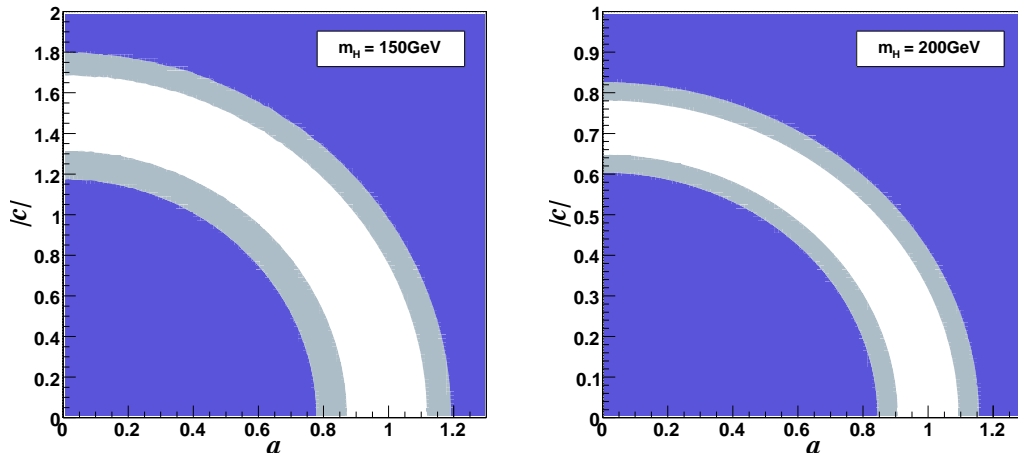
Figure 2: The number of standard deviations from the SM which can be obtained in the process $gg \to H \to Z^*Z^* \to 4$ *leptons*, as a scan over the $(a, |c|)$ plane. The Higgs mass has been chosen to be $150\,\mathrm{GeV}$ *(left)* and $200\,\mathrm{GeV}$ *(right)*. The white region is where the deviation from the SM is less than $3\,\sigma$; in the light blue/light grey region the deviation is between $3\,\sigma$ and $5\,\sigma$; while for the dark blue/dark grey region the deviation is greater than $5\,\sigma$.

cannot distinguish whether or not any deviation is originating from non-standard values of a or $|c|$, and even if the SM total rate is confirmed, one cannot definitively say that a and c take their SM values since an enhancement in $|c|$ may be compensated by a reduction in a. Also, a non-zero value of b could provoke a similar effect. Indeed, the total rate is not even reliable in distinguishing a CP-even eigenstate from a CP-odd one. Instead, to provide a definitive measurement of CP violation in this coupling, one must explore asymmetries which probe the interference of the CP-even and CP-odd contributions directly.

4 Asymmetries as a probe of CP-violation

As stated above, apart from the terms proportional to a and $\Re e(b)$, all other contributions to the vertex Eq. (2) are odd under CP and/or $\tilde{\mathrm{T}}$ transformations, and their presence implies violations of the corresponding symmetries in the interaction. We exploit this by constructing observables from the 3-momenta of the initial and final state particles with the *same* transformation property under the discrete symmetries as one of these non-SM couplings. The expectation value of the *sign* of such a variable will directly

probe the corresponding coupling coefficient [24].[5] The asymmetry will be proportional to the probed coupling and therefore non-zero only if the corresponding non-SM coupling is present. Furthermore, since these asymmetries are exactly zero for all backgrounds (we neglect interference effects), backgrounds cannot contribute to the asymmetry, except through fluctuations, and it is therefore possible to use less stringent cuts on the signal.

In this section we present various observables and their asymmetries which allow one to probe the real and imaginary parts of the form factors b and c, the latter being indicative of CP violation for simultaneously non-zero a and/or b values.

1. An observable to probe $\Im m(c)$: We consider the observable

$$O_1 \equiv \frac{(\vec{p}_{2Z} - \vec{p}_{1Z}) \cdot (\vec{p}_{3H} + \vec{p}_{4H})}{|\vec{p}_{2Z} - \vec{p}_{1Z}||\vec{p}_{3H} + \vec{p}_{4H}|} \ . \tag{11}$$

Here $\vec{p}_i$, $i = 1, \ldots 4$ are the 3-momenta of the leptons (in the order $f_1 \bar{f}_1 f_2 \bar{f}_2$), and the subscripts Z and H denote that the corresponding 3-vector is taken in the Z boson or Higgs boson rest frame, respectively. This observable is CP odd and $\tilde{\mathrm{T}}$ even and thus probes the non-SM coupling with the same transformation properties, *i.e.* $\Im m(c)$. With the above angular definitions we have

$$O_1 = \cos \theta_1 \ . \tag{12}$$

We can calculate the resulting asymmetry by integrating Eq. (3) over the angles with an appropriate weighting. Although Eq. (3) is only valid for distinguishable fermions, we may include fermions of the same flavour, e.g. $(e^- e^+)(e^- e^+)$, and distinguish the fermions by the requirement that the first pair reconstruct the Z-boson mass. In general, the contribution from the same final state with the antiparticles switched would contain two off-shell Z-bosons and may be neglected. However, one should also note that this observable requires one to distinguish between fermions and anti-fermions.

The angular distribution of Eq. (3) contains several terms linear in $\cos \theta_1$. However, most of these terms are removed by integration over the angles θ_2 and ϕ, leaving only one term proportional to $a \Im m(c)$. So only a non-zero value of $\Im m(c)$ gives rise to this forward-backward asymmetry and hence provides a definitive signal of CP violation in the HZZ vertex. This is demonstrated in Fig. 3, which shows the dependence on $\cos \theta_1$ for pure CP-even, pure CP-odd and CP-violating interactions[6].

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[5]This statement is true strictly when only the linear terms in the anomalous HZZ coupling are kept. Potentially, the asymmetries may also contain combinations of more than one (small) anomalous couplings which will have the same discrete symmetry transformation properties. In that case the asymmetry will
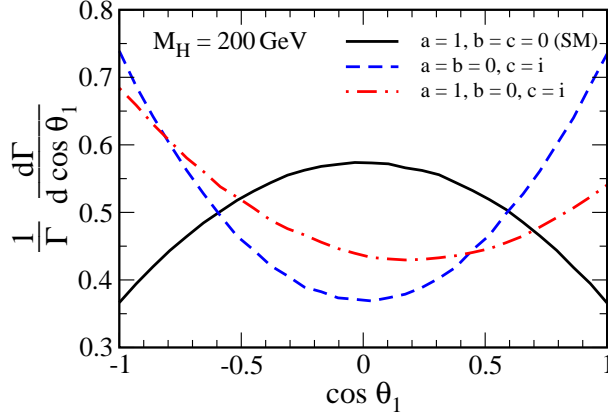
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Figure 3: The normalized differential width for $H \to ZZ \to (f_1 \bar{f}_1)(f_2 \bar{f}_2)$ and $m_H = 200$ GeV with respect to the cosine of the fermion f_1's polar angle θ_1. The solid (black) curve shows the SM case ($a = 1$, $b = c = 0$) while the dashed (blue) curve is for a pure CP-odd state ($a = b = 0$, $c = i$). The dot-dashed (red) curve is for a state with a CP violating coupling ($a = 1$, $b = 0$, $c = i$). One can clearly see an asymmetry about $\cos\theta_1 = 0$ for the CP violating case.

To quantify the effect we define an asymmetry by,

$$\mathcal{A}_1 = \frac{\Gamma(\cos\theta_1 > 0) - \Gamma(\cos\theta_1 < 0)}{\Gamma(\cos\theta_1 > 0) + \Gamma(\cos\theta_1 < 0)} . \tag{13}$$

This asymmetry, which is the expectation value of the sign of $\cos\theta_1$ (Eq. 12) and which is CP-odd and $\tilde{\mathrm{T}}$ even, directly probes $\Im m(c)$ which is also CP-odd and $\tilde{\mathrm{T}}$ even. Integrating Eq. (3), the asymmetry $\mathcal{A}_1$ can be written as

$$\mathcal{A}_1 = \frac{1}{\tilde{\Gamma}} \int d^2\mathcal{P}\,\beta \left\{-3\,a\Im m(c)x\,\eta_1\,\gamma_b\right\}, \tag{14}$$

where $\tilde{\Gamma}$ is related to the decay width $H \to ZZ^{(*)} \to (f_1 \bar{f}_1)(f_2 \bar{f}_2)$, *c.f.* Eqs. (8,9), and is given by

$$\tilde{\Gamma} = \int d^2\mathcal{P}\,\beta \left\{a^2\left(1 + \frac{\gamma_a^2}{2}\right) + |b|^2\frac{\gamma_b^4}{2}x^2 + 4|c|^2x^2\gamma_b^2 + a\Re e(b)x\gamma_a\gamma_b^2\right\}, \tag{15}$$

and the integral is over the virtualities, weighted with the Breit-Wigner form of the Z-boson

be a direct probe of that particular combination of the non-SM couplings.

[6]This figure differs from the corresponding figure in Ref. [53] for the CP violating coupling due to the different conventions. The corresponding curve for the mixed CP state in Ref. [53] is reproduced with our current conventions if $a = 1, b = 0, c = -i/2$.

propagators,

$$\int d^2\mathcal{P}\,\cdots = \int_0^{m_H^2} dm_1^2 \int_0^{[m_H-m_1]^2} dm_2^2 \, \frac{m_1^2}{[(m_1^2-m_Z^2)^2+m_Z^2\Gamma_Z^2]} \frac{m_2^2}{[(m_2^2-m_Z^2)^2+m_Z^2\Gamma_Z^2]}\cdots. \tag{16}$$

This asymmetry is calculated at tree-level. Higher order electroweak corrections to the decay $H \to ZZ \to 4$ leptons are of the order 5-10% for angular distributions [62,63]. One might worry that these corrections could feed into the asymmetry and swamp the signal. However, unless the corrections introduce some new effect (and are thus in some sense "leading order"), one expects their contribution to CP violation to be of a similar proportion as those at tree-level, so they would provide a correction to Eq. (14) of 5-10%, and not significantly alter our results.

Fig. 4 shows the values of $\mathcal{A}_1$ for a Higgs mass of 150 and 200 GeV, respectively, as a function of the ratio $\Im m(c)/a$ and where we have set $b = 0$ for simplicity. The value
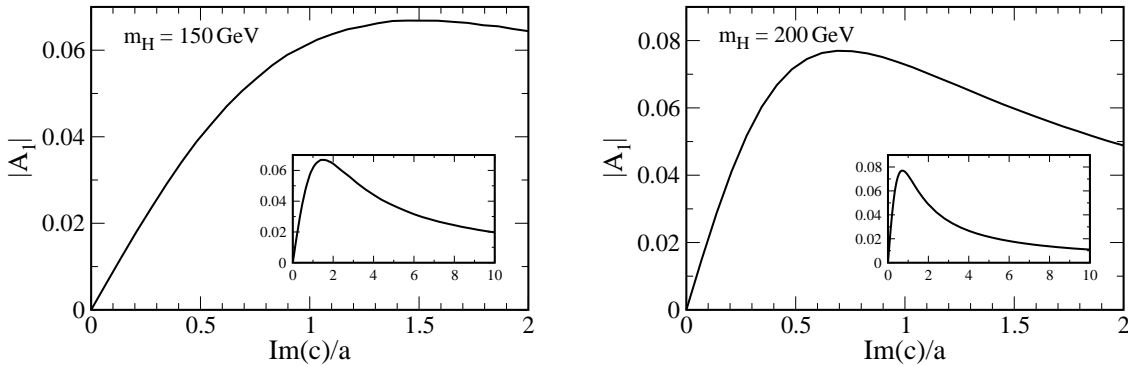

Figure 4: The asymmetry $\mathcal{A}_1$ given by Eq. (14) as a function of the ratio $\Im m(c)/a$, for a Higgs boson of mass 150 GeV (*left*) and 200 GeV (*right*). We chose $b = 0$. The inserts show the same quantities for a larger range of $\Im m(c)/a$.

$\Im m(c)/a = 0$ corresponds to the purely scalar state and $\Im m(c)/a \to \infty$ to the purely CP-odd case. It is clear from Eq. (14) that $\mathcal{A}_1$ is sensitive only to the relative size of the couplings since any overall factor will cancel in the ratio, *c.f.* Eq. (13). We find that the asymmetry is maximal for $\Im m(c)/a \sim 1.5(0.7)$ with a value of about $0.067(0.077)$ for $m_H = 150(200)$ GeV. The smallness of this asymmetry arises from the fact that it is proportional to the coupling $\eta_1 = 2v_1 a_1/(v_1^2 + a_1^2)$ which is equal to approximately 0.149 for e, μ final states, *c.f.* Eq. (14).

In order to estimate whether this asymmetry can be measured at the LHC, we calculate the significance with which a particular CP violating coupling would manifest. To do this, we must take into account the backgrounds to the signal process, which will contaminate

the asymmetry in two ways. Firstly, despite being CP-conserving the backgrounds may contribute to the numerator of the asymmetry via statistical fluctuations (e.g. the background events with $O_1 > 0$ may fluctuate upwards while those with $O_1 < 0$ may fluctuate downwards and vice versa). Secondly, they will directly contribute to the denominator of the asymmetry.

Consequently, the measured asymmetry will be given by,

$$\mathcal{A}_1^{\text{meas}} = \frac{N_S^{\text{asym}}}{N_S + N_B} = \mathcal{A}_1 \frac{N_S}{N_S + N_B}, \tag{17}$$

where N_S^{asym} is the asymmetry in the number of events in the two hemispheres, and $\mathcal{A}_1$ is the perfect theoretical asymmetry given in Eq. (13).

The statistical fluctuation in an asymmetry calculated using a total number of events $N = N_B + N_S$, even when N_B and N_S are expected to be symmetric, is $1/\sqrt{N}$. Hence, the significance of the expected asymmetry, S, in units of this statistical fluctuation is given by

$$S = \mathcal{A}_1^{\text{meas}} \sqrt{N} = \frac{N_S^{\text{asym}}}{\sqrt{N}} = \mathcal{A}_1 \frac{N_S}{\sqrt{N}} . \tag{18}$$

In order to calculate this, we need to know the number of signal and background events expected at the LHC. However, in this case, since the contamination of the significance from the background is rather minimal, we choose to use the event sample *before* the detailed cuts to remove backgrounds, but after the initial selection cuts. For 150 GeV we take the number of signal and background events before applying the additional isolation and impact parameter cuts to remove the irreducible backgrounds, and for 150 GeV we do not apply the final p_T cut on the hardest Z-boson (see Refs.[7,45]).

Then, according to Refs.[7,45], for a $m_H = 150$ GeV SM Higgs boson, we have a signal cross-section of 5.53 fb, with an overall lepton efficiency of 0.7625. Assuming an integrated luminosity of 300 fb^{-1} this gives 1265 signal events. For $m_H = 200$ GeV, the corresponding signal is 1340 events. The number of signal events for the CP violating case is then obtained by multiplying the number of SM events by the ratio of CP violating to SM branching ratios. In the CP-violating case we always assume the SM value for the CP-even coefficient, $a = 1$. For simplicity we assume the charge of the particles to be unambiguously determined, and pair the leptons by requiring at least one pair to reconstruct the Z boson mass. The number of background events before cuts has been derived correspondingly from the study Refs.[7,45] and amounts to 1031(740) events for $m_H = 150(200)$ GeV.

The significances are shown in Figs. 5 for $m_H = 150$ and 200 GeV, respectively, as a function of $\Im m(c)$ with $a = 1$ and $b = 0$. As can be inferred from the figures the maximum of the curves is slightly shifted to higher values of $\Im m(c)/a$ compared to the corresponding
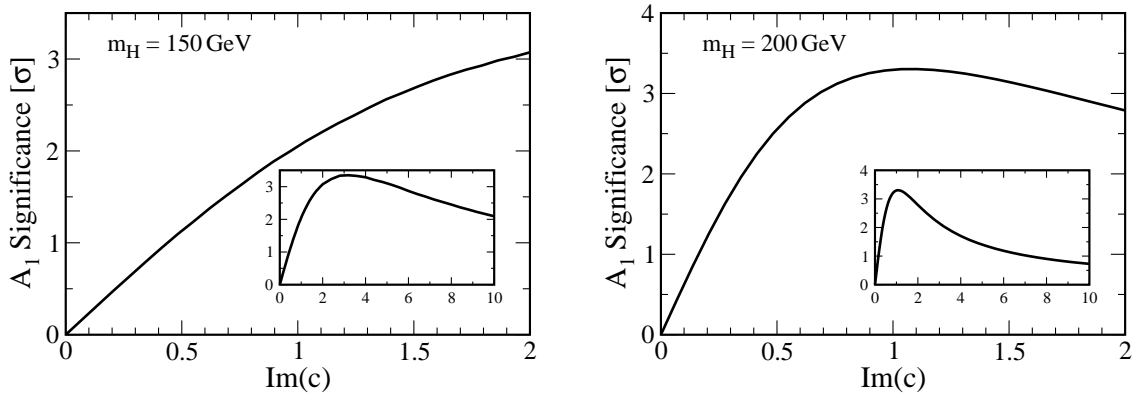
Figure 5: The significances corresponding to the asymmetry $\mathcal{A}_1$ as a function of $\Im m(c)$, for a Higgs boson of mass $150\,\mathrm{GeV}$ (*left*) and 200 GeV (*right*). We chose the CP-even coupling coefficient $a = 1$ and $b = 0$. The inserts show the same quantities for a larger range of $\Im m(c)$.

Figs. 4. This is due to the increasing Higgs decay rate with rising pseudoscalar coupling. The curves show that, even in a best case scenario, the significance is always $\lesssim 3.5\,\sigma$. This asymmetry may provide only *evidence* for CP violation (i.e. a greater than $3\,\sigma$ deviation from the SM) if $\Im m(c) \gtrsim 1.9(0.7)$ for $m_H = 150(200)\,\mathrm{GeV}$.

However, since one does not need to distinguish f_2 and $\bar{f}_2$ one could also consider using jets instead of muons, i.e. $H \rightarrow ZZ \rightarrow l^+l^-jj$, to increase the statistics. If we use the $b\bar{b}$ final state, one can benefit from the increase by a factor ~ 4.5 in the branching ratio of the Z boson into a $b\bar{b}$ pair relative to the branching ratio into a lepton pair. As a matter of fact a study by ATLAS [68] shows that for a Higgs boson mass of 150 GeV with $30\,\mathrm{fb}^{-1}$ it is possible to have a Higgs signal with a significance of 2.7σ in this channel. So indeed one can foresee the use of this channel to add to the sensitivity.

2. Observables which probe $\Re e(c)$ and/or $\Re e(b^*c)$: We have constructed several observables which allow one to probe $\Re e(c)$. For this we need an observable which is CP odd and $\tilde{\mathrm{T}}$ odd. One possible observable is given by

$$O_2 = \frac{(\vec{p}_{2Z} - \vec{p}_{1Z}) \cdot (\vec{p}_{4H} \times \vec{p}_{3H})}{|\vec{p}_{2Z} - \vec{p}_{1Z}||\vec{p}_{4H} \times \vec{p}_{3H}|} \ , \tag{19}$$

which in terms of the scattering angles reads

$$O_2 \equiv -\sin\phi \sin\theta_1 \ . \tag{20}$$

(Since $\sin\theta_1$ is always positive, one could equivalently use $\sin\phi$ as the observable and obtain the same results.) By comparing this angular dependence with the differential

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angular decay width given in Eq. (3), one can see that the corresponding asymmetry should pick up the third term $\sim \eta_1 \eta_2$ of the contribution multiplied with $a\Re e(c)$ and the second term of the contribution multiplied with $\Re e(b^*c)$ and which also contains $\eta_1 \eta_2$. And
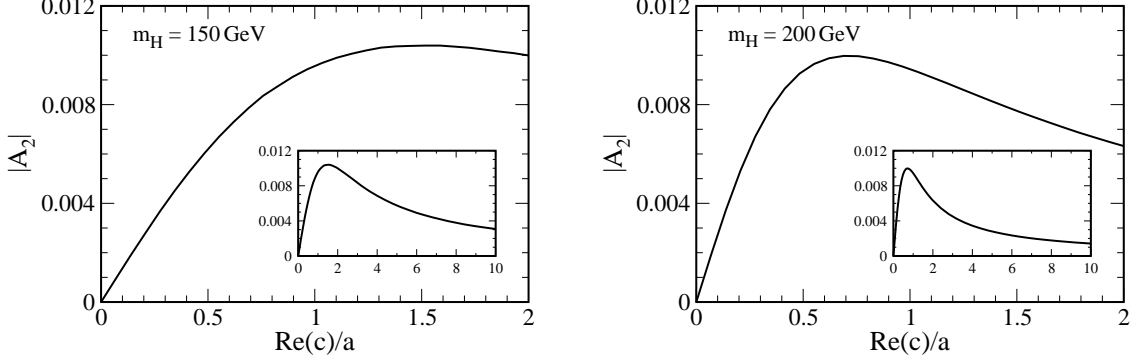

Figure 6: The asymmetry $\mathcal{A}_2$ given by Eq. (21) as a function of the ratio $\Re e(c)/a$, for a Higgs boson of mass $150\,\mathrm{GeV}$ (*left*) and 200 GeV (*right*). We chose $b = 0$. The inserts show the same quantities for a larger range of $\Re e(c)/a$.

indeed we find for this asymmetry

$$
\begin{aligned}
\mathcal{A}_2 &= \frac{\Gamma(O_2 > 0) - \Gamma(O_2 < 0)}{\Gamma(O_2 > 0) + \Gamma(O_2 < 0)} \\
&= \frac{1}{\tilde{\Gamma}} \int d^2 \mathcal{P} \left(\frac{-9\pi}{16} \right) \eta_1 \eta_2 x \gamma_b \left[a\Re e(c)\, \gamma_a + \Re e(b^*c)\, x \gamma_b^2 \right] .
\end{aligned}
\tag{21}
$$

By construction, for $b = 0$ or to linear order in the anomalous couplings, it is proportional to $\Re e(c)$ as expected. This asymmetry is plotted in Figs. 6 as a function of $\Re e(c)/a$ for $m_H = 150$ and 200 GeV, respectively. Since the form factors b, c are expected to be small we do not expect terms of second order in these coefficients to have a large impact, so here and in the following we set $b = 0$. Indeed, for the asymmetry $\mathcal{A}_2$ with $\Re e(b^*c) \approx \Re e(c)^2 \lesssim 0.5$ the change in the asymmetry due to neglecting b is $\lesssim 30\%$. Figs. 6 show that this asymmetry is very small, with values below about ~ 0.011, which is principally due to the proportionality to the small quantity $\eta_1 \eta_2$ in Eq. (21). The significances for the asymmetry $\mathcal{A}_2$ are shown in Figs. 7 for the two Higgs boson mass values. With values below about 0.55 they are far too small to provide evidence for CP-violation due to non-zero $\Re e(c)$. Furthermore, in this case one cannot exploit the decay of Higgs bosons to jets since one must also distinguish $\vec{p}_{3H}$ and $\vec{p}_{4H}$.

The smallness of the asymmetries $\mathcal{A}_1$ and $\mathcal{A}_2$ are directly due to their proportionality to the factors η_1, η_2. Looking at Eq. (3), one sees that this is true for all terms proportional to $a \Im m(c)$, so not much can be done to improve on $\mathcal{A}_1$. However, this is not the case
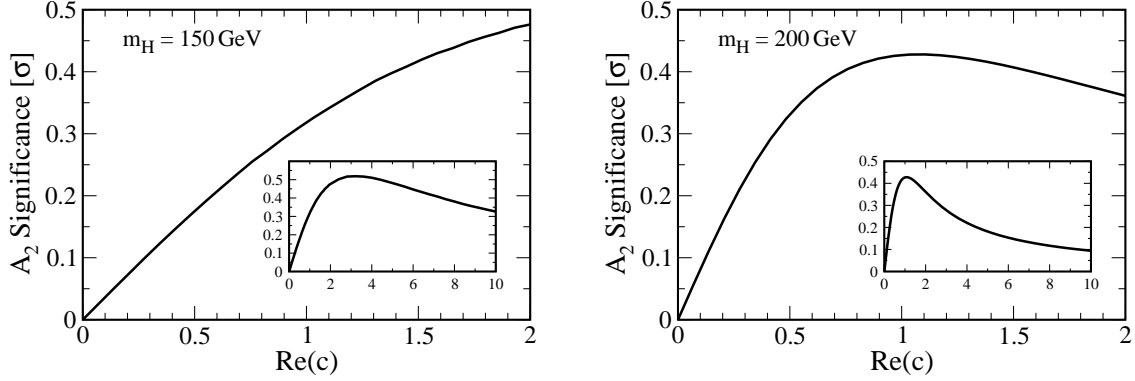
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Figure 7: The significances corresponding to the asymmetry $\mathcal{A}_2$ as a function of $\Re e(c)$, for a Higgs boson of mass $150\,\mathrm{GeV}$ (*left*) and 200 GeV (*right*). We chose the other coupling coefficients $a = 1$ and $b = 0$. The inserts show the same quantities for a larger range of $\Re e(c)$.

for terms proportional to $a\,\Re e(c)$. So we may take our cue from the explicit analytical expression to construct new observables for which the asymmetry will not have these suppression factors. One such observable is given in terms of the angles by

$$O_3 = \cos\theta_1 \sin\theta_2 \cos\theta_2 \sin\phi \ . \tag{22}$$

O_3 can be rewritten using the definition of O_1, *c.f.* Eq. (11), in terms of the four three-vectors,

$$O_3 = O_1\,O_{3a}\,O_{3b} \ , \tag{23}$$

where

$$
\begin{aligned}
O_{3a} &= \frac{(\vec{p}_{4Z} - \vec{p}_{3Z}) \cdot (\vec{p}_{1H} \times \vec{p}_{2H})}{|\vec{p}_{4Z} - \vec{p}_{3Z}||\vec{p}_{1H} \times \vec{p}_{2H}|} \ , \\
O_{3b} &= \frac{(\vec{p}_{3Z} - \vec{p}_{4Z}) \cdot (\vec{p}_{1H} + \vec{p}_{2H})}{|\vec{p}_{3Z} - \vec{p}_{4Z}||\vec{p}_{1H} + \vec{p}_{2H}|} \ .
\end{aligned}
\tag{24}
$$

In order to exploit this observable, we have to discriminate between all four leptons. For the asymmetry $\mathcal{A}_3$,

$$\mathcal{A}_3 = \frac{\Gamma(O_3 > 0) - \Gamma(O_3 < 0)}{\Gamma(O_3 > 0) + \Gamma(O_3 < 0)}, \tag{25}$$

we find analytically

$$\mathcal{A}_3 = \frac{1}{\tilde{\Gamma}} \int d^2\mathcal{P}\left(\frac{\gamma_b x}{\pi}\right)\left[a\Re e(c)\,\gamma_a + \Re e(b^* c)\,x\gamma_b^2\right] \ . \tag{26}$$

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Note that it no longer contains the suppression factors η_1, η_2 and for $b = 0$ it probes the real part of the form factor c. By comparing the angular structure of O_3 with the differential angular distribution Eq. (3), one sees that the asymmetry $\mathcal{A}_3$ picks up the first term in the contribution proportional to $a\Re e(c)$ and the first term in the one proportional to $\Re e(b^*c)$. A non-zero value of $\mathcal{A}_3$ is hence an unambiguous sign of CP-violation.
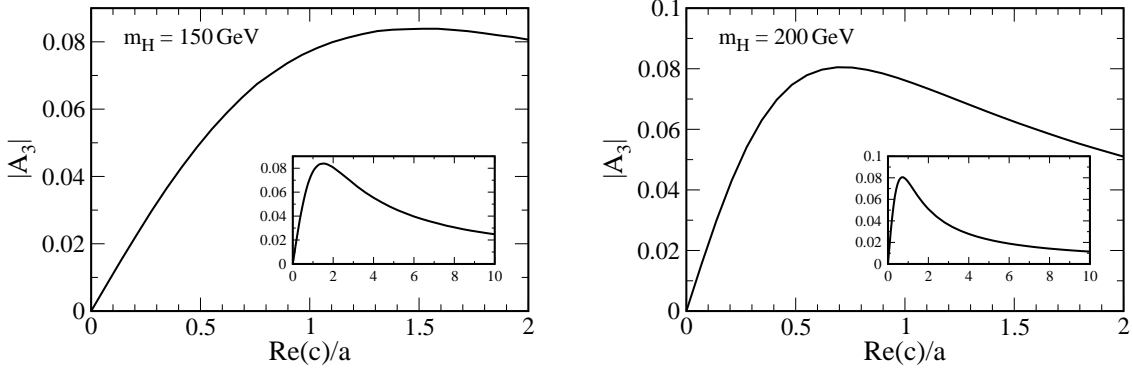

Figure 8: The asymmetry $\mathcal{A}_3$ given by Eq. (26) as a function of the ratio $\Re e(c)/a$, for a Higgs boson of mass $150\,\mathrm{GeV}$ (*left*) and 200 GeV (*right*). We chose $b = 0$. The inserts show the same quantities for a larger range of $\Re e(c)/a$.

Figs. 8 show the asymmetry $\mathcal{A}_3$ for $m_H = 150$ and $200\,\mathrm{GeV}$, respectively, where we have taken $b = 0$ for simplicity. With values of $\lesssim 0.09$ they are about a factor 10 larger than those of $\mathcal{A}_2$. The corresponding significances which should be achievable at the LHC
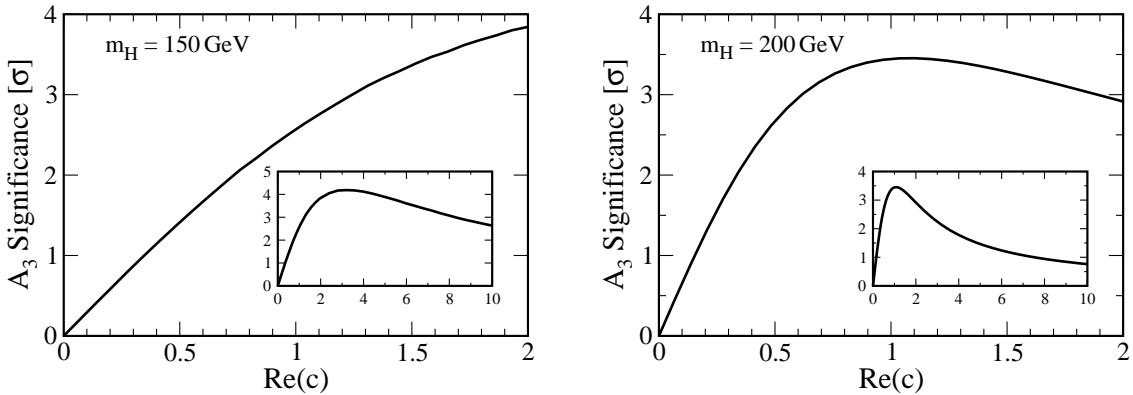

Figure 9: The significances corresponding to the asymmetry $\mathcal{A}_3$ as a function of $\Re e(c)$, for a Higgs boson of mass $150\,\mathrm{GeV}$ (*left*) and 200 GeV (*right*). We chose the other coupling coefficients $a = 1$ and $b = 0$. The inserts show the same quantities for a larger range of $\Re e(c)$.

for this asymmetry are shown for $m_H = 150$ and $200\,\mathrm{GeV}$ in Figs. 9. They are maximal at

$\Re e(c) \approx 3(1)$ for $m_H = 150(200)$ GeV. For a $150\,(200)$ GeV Higgs boson this asymmetry would provide evidence for CP-violation for $\Re e(c) \gtrsim 1.25\,(0.6)$ though discovery (a $5\,\sigma$ deviation) is still out of reach.

One should note, however, that a zero value for this asymmetry does not imply the absence of CP-violation, since for $b \neq 0$ it could also happen that the contributions proportional to $a\Re e(c)$ and $\Re e(b^*c)$ cancel and mimic CP-conservation. In order to unambiguously show CP-violation in the HZZ coupling we hence need an additional observable to determine the two unknowns $\Re e(c)$ and $\Re e(b^*c)$. Such additional observables are presented in the following.

An observable, which probes $\Re e(c)$ alone, is given by

$$O_4 = \frac{[(\vec{p}_{3H} \times \vec{p}_{4H}) \cdot \vec{p}_{1H}][(\vec{p}_{3H} \times \vec{p}_{4H}) \cdot (\vec{p}_{1H} \times \vec{p}_{2H})]}{|\vec{p}_{3H} + \vec{p}_{4H}|^2 |\vec{p}_{1H} + \vec{p}_{2H}||\vec{p}_{3Z} - \vec{p}_{4Z}|^2 |\vec{p}_{1Z} - \vec{p}_{2Z}|^2/16} \ . \tag{27}$$

In terms of the angles it reads

$$O_4 = \sin^2 \theta_1 \sin^2 \theta_2 \sin \phi \cos \phi \ . \tag{28}$$

(Again, since $\sin^2 \theta_{1,2}$ are always positive, this is equivalent to using an observable $\sin 2\phi$.) This coupling structure appears in the decay width only in the contribution which is proportional to $a\Re e(c)$, *c.f.* Eq. (3), so that we can expect the corresponding asymmetry to probe CP-violation due to simultaneous non-vanishing form factors a and c unambiguously. Indeed the asymmetry is given by

$$
\begin{aligned}
\mathcal{A}_4 &= \frac{\Gamma(O_4 > 0) - \Gamma(O_4 < 0)}{\Gamma(O_4 > 0) + \Gamma(O_4 < 0)} \\
&= \frac{1}{\bar{\Gamma}} \int d^2\mathcal{P} \left(\frac{-2}{\pi}\right) a\Re e(c) x \gamma_b \ .
\end{aligned} \tag{29}
$$

Furthermore, as can be inferred from Figs. 10, which show the asymmetry for $m_H = 150$ and 200 GeV as a function of $\Re e(c)/a$, the asymmetries are larger than those of $\mathcal{A}_2$ and $\mathcal{A}_3$, with maximal values of up to ~ 0.11. The significances which may be achieved at the LHC are shown in Figs. 11. They reach values of up to almost 5 for $m_H = 150, 200$ GeV so that this observable may probe CP-violation in an unambiguous way at the LHC for sufficiently large values of $\Re e(c)$. As can be inferred from Figs. 11, for a 150 GeV Higgs boson evidence for a non-zero $\Re e(c)$ is possible for $\Re e(c) \gtrsim 1$, while for a 200 GeV Higgs boson evidence is already possible for $\Re e(c) \gtrsim 0.4$.

Alternatively one may use a combination of O_3 and O_4 to test CP-violation due to non-vanishing $a\Re e(c)$ and/or $\Re e(b^*c)$. For example, in terms of the angles a possible observable
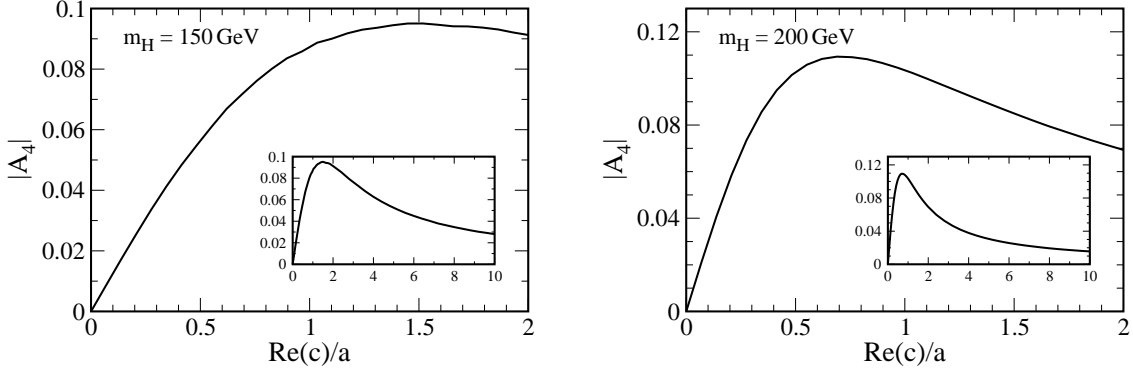
Figure 10: The asymmetry $\mathcal{A}_4$ given by Eq. (29) as a function of the ratio $\Re e(c)/a$, for a Higgs boson of mass $150\,\text{GeV}$ (*left*) and 200 GeV (*right*). The inserts show the same quantities for a larger range of $\Re e(c)/a$.
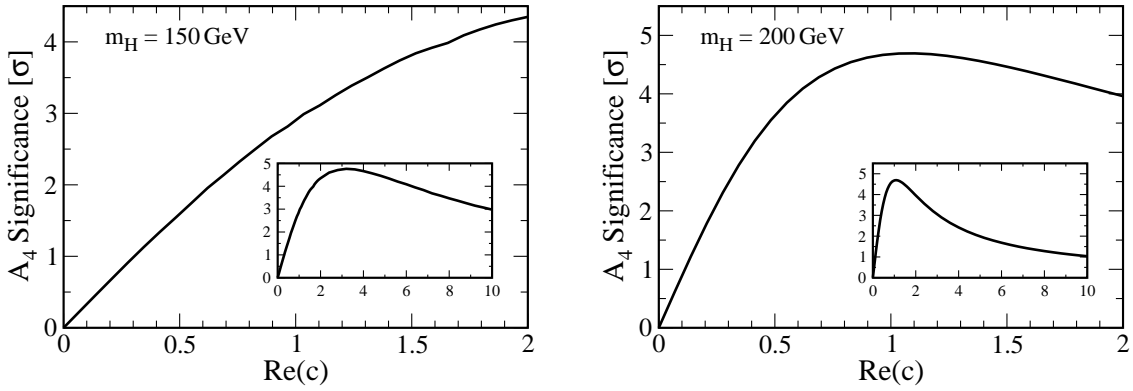

Figure 11: The significances corresponding to the asymmetry $\mathcal{A}_4$ as a function of $\Re e(c)$, for a Higgs boson of mass $150\,\text{GeV}$ (*left*) and 200 GeV (*right*). We chose the other coupling coefficients $a = 1$ and $b = 0$. The inserts show the same quantities for a larger range of $\Re e(c)$.

O_5 is given by

$$O_5 = \sin\theta_1 \sin\theta_2 \sin\phi[\sin\theta_1 \sin\theta_2 \cos\phi - \cos\theta_1 \cos\theta_2] \tag{30}$$

and can be constructed from the three-vectors by

$$O_5 = \frac{[(\vec{p}_{4H} \times \vec{p}_{3H}) \cdot \vec{p}_{1H}][(\vec{p}_{1Z} - \vec{p}_{2Z}) \cdot \vec{p}_{3Z}]}{|\vec{p}_{3H} + \vec{p}_{4H}||\vec{p}_{3Z} - \vec{p}_{4Z}|^2|\vec{p}_{1Z} - \vec{p}_{2Z}|^2/8} \ . \tag{31}$$

The related asymmetry

$$\mathcal{A}_5 = \frac{\Gamma(O_5 > 0) - \Gamma(O_5 < 0)}{\Gamma(O_5 > 0) + \Gamma(O_5 < 0)} \tag{32}$$

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is shown in Figs. 12 for $m_H = 150, 200$ GeV and yields the largest values among the asymmetries discussed so far, up to ~ 0.15. In Figs.13 we show the corresponding significances. We see that for a 150 (200) GeV Higgs boson, this asymmetry would provide evidence for CP-violation for $\Re e(c) \gtrsim 0.66\,(0.25)$ and discovery of CP-violation for $\Re e(c) \gtrsim 1.28(0.52)$.
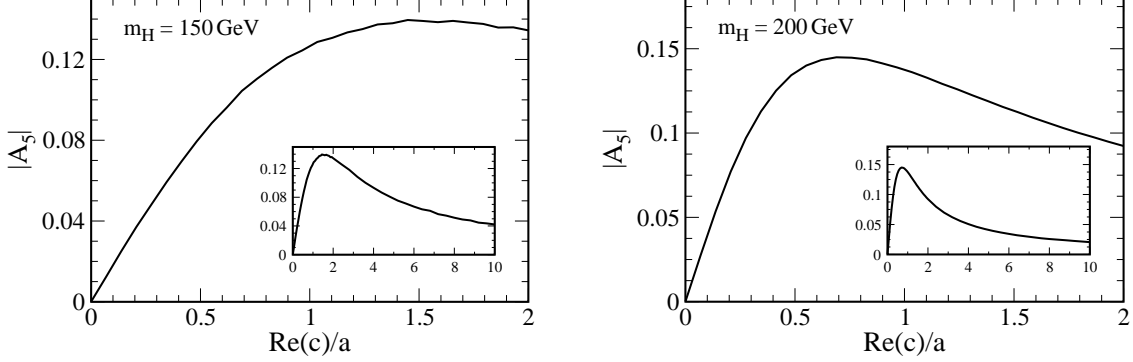

Figure 12: The asymmetry $\mathcal{A}_5$ given by Eq. (32) as a function of the ratio $\Re e(c)/a$, for a Higgs boson of mass 150 GeV (*left*) and 200 GeV (*right*). We chose $b = 0$. The inserts show the same quantities for a larger range of $\Re e(c)/a$.
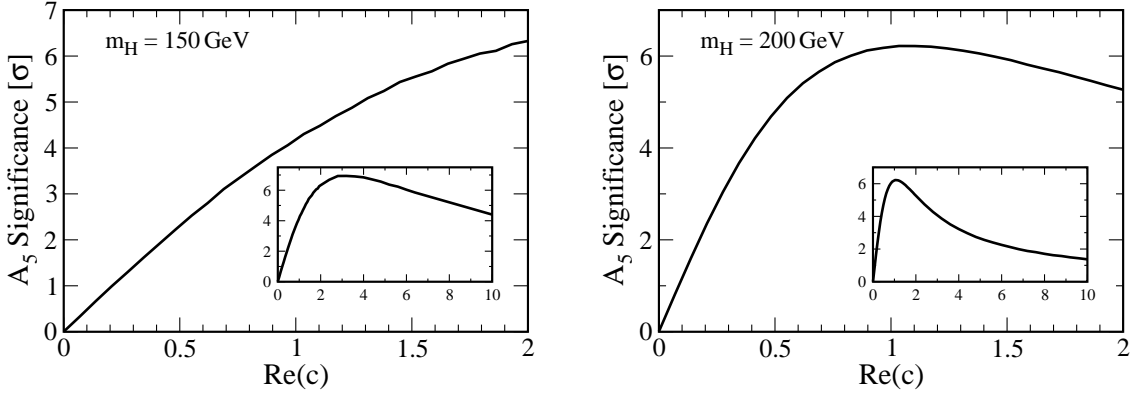

Figure 13: The significances corresponding to the asymmetry $\mathcal{A}_5$ as a function of $\Re e(c)$, for a Higgs boson of mass 150 GeV (*left*) and 200 GeV (*right*). We chose the other coupling coefficients $a = 1$ and $b = 0$. The inserts show the same quantities for a larger range of $\Re e(c)$.

3. An observable which probes $\Im m(b)$: For completeness, we also present an observable that probes the imaginary part of the CP-even form factor b. It is given by the

following combination of three-vectors

$$O_6 = \frac{[(\vec{p}_{1Z} - \vec{p}_{2Z}) \cdot (\vec{p}_{3H} + \vec{p}_{4H})][(\vec{p}_{3H} \times \vec{p}_{4H}) \cdot \vec{p}_{1H}]}{|\vec{p}_{1Z} - \vec{p}_{2Z}|^2|\vec{p}_{3H} + \vec{p}_{4H}|^2|\vec{p}_{3Z} - \vec{p}_{4Z}|/4}$$
$$= \sin\theta_1 \cos\theta_1 \sin\theta_2 \sin\phi . \tag{33}$$

And the asymmetry reads analytically

$$\mathcal{A}_6 = \frac{\Gamma(O_6 > 0) - \Gamma(O_6 < 0)}{\Gamma(O_6 > 0) + \Gamma(O_6 < 0)}$$
$$= \frac{1}{\tilde{\Gamma}} \int d^2\mathcal{P} \frac{3}{8} \eta_2 \, a \Im m(b) \, x\gamma_b^2 . \tag{34}$$

Figs. 14 and 15 show the corresponding asymmetries and significances. Notice that once again, the asymmetry is proportional to the small factor η_2 and is therefore rather small, i.e. $\lesssim 0.025$. Correspondingly this observable does not provide a good significance (only reaching values of about 1), so that the extraction of $\Im m(b)$ from this observable does not seem to be feasible at the LHC. Unfortunately, since this small factor is present in all the relevant terms in Eq. (3), all asymmetries that one can construct to probe this coefficient will be similarly small.
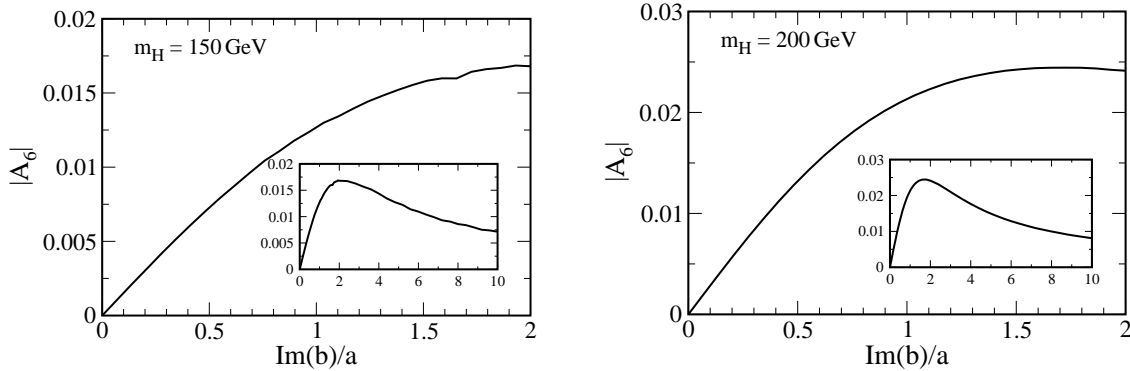

Figure 14: The asymmetry $\mathcal{A}_6$ given by Eq. (34) as a function of the ratio $\Im m(b)/a$, for a Higgs boson of mass $150\,\text{GeV}$ (left) and 200 GeV (right). We chose $c = 0$. The inserts show the same quantities for a larger range of $\Im m(b)/a$.

Refs.[28,29] also consider reweighting observables with the product of the energy differences between the paired leptons, i.e. $(E_2 - E_1)(E_4 - E_3)$. In our notation, this product can be written,

$$(E_2 - E_1)(E_4 - E_3) = \gamma_1\gamma_2\beta_1\beta_2 m_1 m_2 \cos\theta_1 \cos\theta_2 . \tag{35}$$

So this procedure places more importance on events with highly boosted Z bosons and/or events where the lepton is emitted along the line of the parent Z boson's direction of
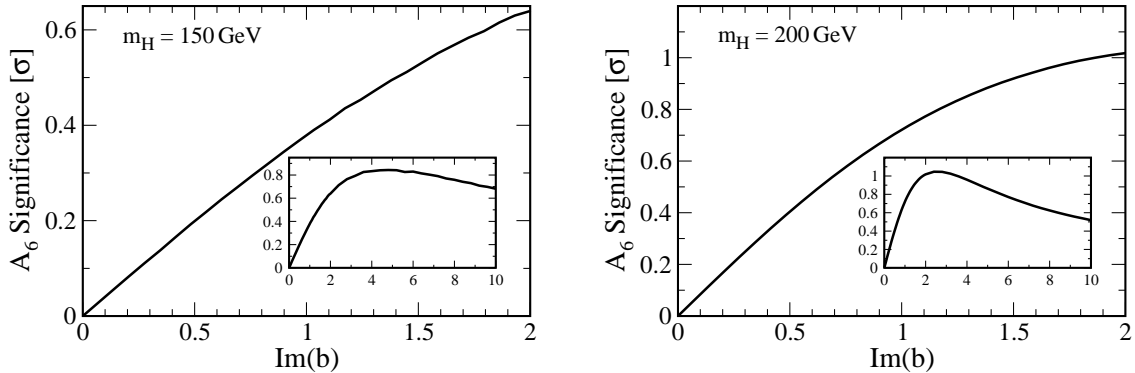
Figure 15: The significances corresponding to the asymmetry $\mathcal{A}_6$ as a function of $\Im m(b)$, for a Higgs boson of mass 150 GeV (*left*) and 200 GeV (*right*). We chose the other coupling coefficients $a = 1$ and $c = 0$. The inserts show the same quantities for a larger range of $\Im m(b)$.

travel. Only the latter would affect our asymmetries and is similar (in principle and effect) to making a different choice of the observable, O_i. Thus, the procedure adopted by these authors is analogous to what we have done.

In summary, using the observables O_1, O_3, O_4 and O_5 and their corresponding asymmetries at the LHC, we can in principle place limits on (or provide evidence for) CP-violation due to the simultaneous presence of CP-even and CP-odd form factors. For both a 150 GeV and 200 GeV Higgs boson, O_1, O_3, O_4 and O_5 all provide evidence (and O_5 potential discovery) for some values of the additional non-SM couplings. Unfortunately, O_2 and O_6 are rather insensitive (due to the requirement of vector-axial interference) and cannot be used to place useful limits on additional couplings. The observables O_3 and O_5 can not unambiguously rule out CP-violation, since their dependence also on $\Re e(b^*c)$ may provide an accidental cancellation with the terms proportional to $a\Re e(c)$. However, O_4 only depends on $a\Re e(c)$ and can thus test violation of the CP quantum numbers due to non-vanishing a and c. With the three observables $O_{3,4,5}$ at hand we can furthermore also extract the value of $\Re e(b^*c)$ and finally do consistency tests as well as reduce the effect of experimental errors.

From a theoretical perspective, these asymmetries (if measurable) are sufficient to determine all the form factors a, b and c of our general CP-violating HZZ coupling, with real and imaginary parts. This is summarized in Table 1, which shows the various dependencies of the described observables on the form factors. We have 6 observables for the five unknowns a, $\Re e(b)$, $\Im m(b)$, $\Re e(c)$ and $\Im m(c)$. If we furthermore assume that any

Asymmetry/form factor	a	$\Re e(b)$	$\Im m(b)$	$\Re e(c)$	$\Im m(c)$
$\mathcal{A}_1$	x				x
$\mathcal{A}_2$	x	(x)	(x)	x	(x)
$\mathcal{A}_3$	x	(x)	(x)	x	(x)
$\mathcal{A}_4$	x			x	
$\mathcal{A}_5$	x	(x)	(x)	x	(x)
$\mathcal{A}_6$	x		x		

Table 1: The dependence of the asymmetries $\mathcal{A}_1$ to $\mathcal{A}_6$ on the form factors a, b, c of the general HZZ coupling Eq. (2). (x) denotes a dependence which is suppressed if the additional form factors are small.

product of b and c is very small (if b and c are loop suppressed or suppressed by some scale of new physics) we may neglect their simultaneous influence in the asymmetries $\mathcal{A}_2$, $\mathcal{A}_3$ and $\mathcal{A}_5$ and only require two of the asymmetries $\mathcal{A}_3$ to $\mathcal{A}_5$ ($\mathcal{A}_2$ not being of much use due to its smallness) to extract a and $\Re e(c)$, while relying on $\mathcal{A}_1$ and $\mathcal{A}_6$ for $\Im m(c)$ and $\Im m(b)$ respectively. However, the analysis done here and the smallness of the asymmetries implies that only $\Re e(c)$, and possibly $\Im m(c)$, are likely to be seen if non-zero. Also note that many of these asymmetries are highly correlated with one another.

Of course, the final feasibility of detecting or placing limits on non-SM form factors depends on the real experimental environment. We simulated this here by taking the values given by the ATLAS studies. Any further refinement is beyond the scope of this study, but we have shown here the utility of these observables in providing unambiguous information on possible non-SM terms in the HZZ coupling and the consequent CP-violation. Any evidence or discovery of CP-violation crucially depends on the size of the non-SM form factors. Irrespective of this one may use these observables to place experimental limits on their values.

5 Kinematical distributions as a probe of CP-violation

As we have seen, the asymmetries discussed in section 4 are most useful for a Higgs boson with mass $m_H \gtrsim 2m_Z$. For a lighter Higgs boson the rates are much smaller and

the significance may not be sufficient for identifying CP-violation or setting satisfactory limits. In this case, one must rely on fitting shapes of kinematic distributions that depend on the CP character of the Higgs boson. From the discussions in the literature it is clear that the angle ϕ between the planes of the two fermion pairs coming from the decays of the Z bosons, and the polar angle of the fermions f_1 or f_2 in the rest frames of the Z bosons, θ_i $(i = 1, 2)$, are suitable variables [30] (see Fig. 1).

1) The angular distribution in ϕ: In the decay process Eq. (1), let us consider the azimuthal angular distribution $d\Gamma/d\phi$. Integrating Eq. (3) over θ_1, θ_2 and taking a CP-violating coupling with a and c non-zero[7] we find

$$\frac{d\Gamma}{d\phi} \sim b_1 + b_2 \cos\phi + b_3 \sin\phi + b_4 \cos 2\phi + b_5 \sin 2\phi , \qquad (36)$$

where b_i $(i = 1, ..., 5)$ are functions of m_H and m_Z in terms of γ_a, γ_b,

$$\begin{aligned}
b_1 &= a^2(2 + \gamma_a^2) + 8|c|^2 x^2 \gamma_b^2 \\
b_2 &= -\frac{9\pi^2}{32} a^2 \eta_1 \eta_2 \gamma_a \\
b_3 &= \frac{9\pi^2}{16} a\Re e(c) \eta_1 \eta_2 x \gamma_a \gamma_b \qquad (37) \\
b_4 &= \frac{a^2}{2} - 2|c|^2 x^2 \gamma_b^2 \\
b_5 &= -2a\Re e(c) x \gamma_b .
\end{aligned}$$

Whereas the purely SM case $(a = 1, b = c = 0)$ shows a distribution (see also Ref. [12])

$$\begin{aligned}
\frac{d\Gamma}{d\phi} &\sim 1 + a_2 \cos\phi + a_4 \cos 2\phi , \\
a_2 &= -\frac{9\pi^2}{32}\eta_1\eta_2\frac{\gamma_a}{2 + \gamma_a^2} \qquad (38) \\
a_4 &= \frac{1}{2}\frac{1}{2 + \gamma_a^2} ,
\end{aligned}$$

in the purely pseudoscalar case $(a = b = 0, c \neq 0)$ we have

$$\frac{d\Gamma}{d\phi} \sim 1 - \frac{1}{4}\cos 2\phi . \qquad (39)$$

In the CP violating case the inclusion of contributions from both the scalar and pseudoscalar couplings alters the angular behaviour via the occurrence of $\sin\phi$ and $\sin 2\phi$ terms, and a reweighting of the other terms. Knowing the Higgs mass from previous measurements, any deviation from the predicted distribution in the purely scalar/pseudoscalar case will be indicative of CP violation. This can be inferred from Fig. 16 which shows the

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[7]The expression with all three coupling coefficients a, b and c non-zero is given in the Appendix.
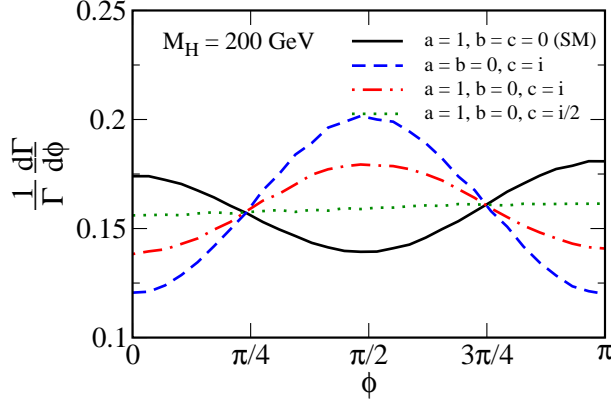
Figure 16: The normalized differential width for $H \to Z^{(*)}Z \to (f_1\bar{f}_1)(f_2\bar{f}_2)$ with respect to the azimuthal angle ϕ. The solid (black) curve shows the SM case ($a = 1$, $b = c = 0$) while the dashed (blue) curve is a pure CP-odd state ($a = b = 0$, $c = i$). The dot-dashed (red) curve and the dotted (green) curve are for states with CP violating couplings $a = 1$, $b = 0$ with $c = i$ and $c = i/2$, respectively.

azimuthal angular distribution for $m_H = 200\,\text{GeV}$ in the SM case, for a purely CP-odd Higgs boson and for two CP violating cases. The purely CP-odd curve will always show the same behaviour independently of the value of c since the curves are normalized to unit area. Therefore a special value of c could not fake the flattening of the curve appearing in the CP violating examples. This flattening even leads to an almost constant distribution in ϕ for the case $c/a = i/2$. It should be kept in mind though, that this method cannot be applied for very large Higgs masses where the ϕ dependence is washed out. One must also beware of degenerate Higgs bosons of opposite CP; since the decay products are the same, they will both contribute to the rate and must be summed coherently, possibly mimicking the effect seen above.

This procedure is similar to that of Refs. [51,52] where log-likelihood functions were constructed and minimised to extract the coefficients in the vertex or yield exclusion contours.

2) The angular distribution in θ_i: Integrating Eq. (2) over ϕ and $\cos\theta_2$ provides a distribution in $\cos\theta_1$. For the CP violating case $a, c \neq 0$, $b = 0$ we find,

$$\frac{d\Gamma}{d\cos\theta_1} \sim a^2\left[(\gamma_a^2 - 1)\sin^2\theta_1 + 2\right] + 4|c|^2 x^2 \gamma_b^2 (1 + \cos^2\theta_1) - 8a\Im m(c)\eta_1 x \gamma_b \cos\theta_1. \tag{40}$$

In the purely SM case we recover,

$$\frac{d\Gamma}{d\cos\theta_1} \sim \sin^2\theta_1 + \frac{2}{\gamma_a^2 - 1}, \tag{41}$$

27

which for large Higgs boson masses ($\gamma_a \to \infty$) reproduces the well-known behaviour $\sim \sin^2 \theta_1$. In contrast, in the purely CP odd case we have

$$\frac{d\Gamma}{d\cos\theta_1} \sim 1 + \cos^2\theta_1 \ . \tag{42}$$

CP violation is manifest by a linear dependence on $\cos\theta_1$. However, due to the proportionality to η_1 the CP violating effect in the angular distribution is small, which is reflected also in the smallness of the asymmetry $\mathcal{A}_1$. See also the discussion in Section 4 and Fig. 3.

2) The threshold distribution: In principle, information about the form factors of the HZZ vertex is also encoded in the dependence of its partial width on the virtuality of the Z-bosons [30]. In particular, looking at Eq. (9) one sees that only the term proportional to a^2 contains a linear dependence in β. This is due to there being no momentum dependence in the SM HZZ vertex, in contrast to the additional non-SM terms of Eq. (2); the single β arises from the phase space. Consequently, one can distinguish a CP-even Higgs boson from a CP-odd Higgs boson decaying to ZZ^* by examining the threshold behaviour since the CP-even excitation curve will be much steeper. This is illustrated in Fig. 17 where one can clearly see the steeper dependence on the virtuality M^* of the most off-shell Z-boson for the CP-even case compared to the CP-odd case.
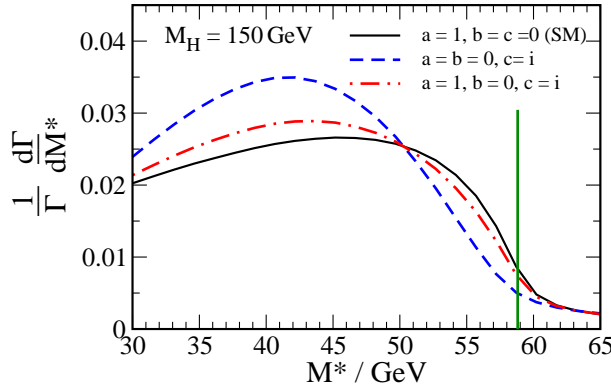

Figure 17: The normalized differential width for $H \to Z^{(*)}Z \to (f_1\bar{f}_1)(f_2\bar{f}_2)$ with respect to the virtuality of the (most) off-shell Z-boson M^*. The solid (black) curve shows the SM case ($a = 1$, $b = c = 0$) while the dashed (blue) curve is a pure CP-odd state ($a = b = 0$, $c = i$). The dot-dashed (red) curve is for states with CP violating couplings $a = 1$, $b = 0$ with $c = i$. The vertical green line represents the nominal threshold at $m_H - m_Z$.

However, this behaviour near threshold will be dominated by whichever term has the lowest power of β. So when one has a Higgs boson of mixed CP, the SM term will always dominate at threshold. This is also shown in Fig. 17 where the curve for the CP-violating

case sits almost on top of the SM curve near threshold. So while the threshold dependence is very good at distinguishing a pure CP-even Higgs boson from a pure CP-odd one, it is unfortunately not very helpful for distinguishing a CP-violating Higgs from the SM case.

6 Conclusions

In this work we have studied the process $H \rightarrow ZZ^{(*)} \rightarrow 4l$, $(l = e, \mu)$ at the LHC to determine how well a general CP violating HZZ coupling can be tested.

We examined the dependence of the partial width on non-SM form factors. By making use of the expected numbers of SM signal and background events, after cuts, provided by the ATLAS experiment, we produced exclusion plots for these non-SM form factors. We demonstrated that while large non-SM form factors may cause large deviations, it is difficult to distinguish their effect from an enhanced (or diminished) SM coupling.

We then presented asymmetries which are non-vanishing when non-SM form factors are present in the HZZ coupling. We found a set of observables which, in principle, allows the extraction of the real and imaginary parts of all the complex form factors in the non-SM part of the HZZ vertex, if the significances are large enough. We analysed these asymmetries in the context of the ATLAS $H \rightarrow ZZ^{(*)} \rightarrow 4l$ study, and found that some of these asymmetries may be large enough to provide evidence of CP violation and in some cases even discovery, depending of course on the specific values of the CP violating contributions. In any case, these asymmetries will be useful in putting limits on any possible extra HZZ couplings beyond the tree-level SM, and deserve further experimental analysis.

Furthermore, we presented an analytic formula for the partial width with full dependence on the final state azimuthal and polar angles, and demonstrated that the angular distributions may be exploited for Higgs boson masses below the threshold. Indeed, the azimuthal angle between the two decay planes of the Z bosons is sensitive to CP violation if the Higgs boson mass is not too large.

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8 Appendix

For the process $H \to ZZ^{(*)} \to (f_1\bar{f}_1)(f_2\bar{f}_2)$ with a general CP-violating coupling *c.f.* Eq. (2), we present here the differential distribution in the angle ϕ between the planes of the two fermion pairs coming from the decays of the $Z^{(*)}$ bosons, taking into account the full dependence on the form factors a, b and c. The notation is as fixed in the text.

$$\frac{d\Gamma}{d\phi} \sim b_1 + b_2 \cos\phi + b_3 \sin\phi + b_4 \cos 2\phi + b_5 \sin 2\phi \;, \tag{43}$$

where

$$
\begin{aligned}
b_1 &= a^2(2 + \gamma_a^2) + |b|^2 x^2 \gamma_b^4 + 8\,|c|^2\,x^2\gamma_b^2 + 2a\Re e(b)\,x\gamma_a\gamma_b^2 \\
b_2 &= -\frac{9\pi^2}{32}\eta_1\eta_2\left[a^2\gamma_a + a\Re e(b)\,x\gamma_b^2\right] \\
b_3 &= \frac{9\pi^2}{16}\eta_1\eta_2\left[\Re e(b^*c)\,x^2\gamma_b^3 + a\Re e(c)\,x\gamma_a\gamma_b\right] \\
b_4 &= \frac{a^2}{2} - 2\,|c|^2\,x^2\gamma_b^2 \\
b_5 &= -2a\Re e(c)\,x\gamma_b\;.
\end{aligned}
\tag{44}
$$

The polar angular distribution in θ_1 is given by

$$
\begin{aligned}
\frac{d\Gamma}{d\cos\theta_1} &\sim a^2[(\gamma_a^2 - 1)\sin^2\theta_1 + 2] + |b|^2 x^2\gamma_b^4\sin^2\theta_1 + 4|c|^2 x^2\gamma_b^2(1 + \cos^2\theta_1) \\
&+ 2a\Re e(b)x\gamma_a\gamma_b^2\sin^2\theta_1 - 8a\Im m(c)\eta_1 x\gamma_b\cos\theta_1\;.
\end{aligned}
\tag{45}
$$

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